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The gradation puzzle of intellectual assurance

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Epistemic justification typically varies in strength according to whether one perceives a paradigmatic or a marginal instance of the relevant property. During a painful sensation, for example, the justification that one feels pain is weak if the pain is mild; it is strong if the pain is sharp. Cartesians commonly claim that when the pain is sufficiently strong, the justification that one feels

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pain can be infallible. Although 'infallibility' implies the absence of possible errors, the state is epistemically grounded in phenomenal features of one's conscious mind. In particular, the infallible justification that one feels pain cannot be construed without one's awareness of the painful experience. Though once abandoned as 'the mysterious given', infallibility regains currency as a source of 'intellectual assurance' (Ballantyne 2012, Coppenger and Bergmann 2016). Fumerton (2009: 72) specifically contends that infallibility 'gives one complete assurance of the truth'. Motivated by this infallibility thesis and the above observation on gradation, some epistemologists regard infallibility as gradable – for example, two persons can both be infallibly justified regarding their pains, while the one with stronger pain is more justified (Steup 2016, Tucker 2016, Fumerton 2009, 2016, 2018, Ballantyne 2012). Here is the puzzle: if infallible justification varies in strength, how can it always be absolute? This paper explains how the conflict between absoluteness and gradation troubles infallibility. More precisely, by refuting the recent proposal of Fumerton (2016, 2018), we argue that infallibility cannot be salvaged by reducing the gradation to semantic vagueness.

The gradation puzzle can be formulated with the following inconsistent pair of statements.

(Absolute) An infallible justification for *P* leaves absolutely no possibility for error.

(Gradation) An infallible justification for *P* can leave more possibilities for error than another infallible justification for *P*′.

(Absolute) requires infallibility to remove every possibility for error. (Gradation) assumes that infallibility to leaves more or fewer possibilities for error. Thus, the two theses cannot both be true.

Both theses, however, are indispensable for a plausible theory of infallibility. No justification is infallible without (Absolute). (Gradation) is also epistemically motivated: infallibility must be gradable because its epistemic ground, i.e., phenomenal experience, varies in degree. The truth of (Gradation) is obvious when *P* and *P*′ are identical. The infallible justification for the same proposition that one feels pain is obviously gradable according to the pain's intensity. With stronger pains, more *epistemic possibilities of error* are eliminated where one's sensations are similar to but different from pain. In general, the gradation persists when *P* and *P*′ are distinct. The infallible justification of an individual that he feels pain can outperform his infallible justification that he sees red – especially when his pain is more paradigmatic than his red sensation. Apparently, we cannot neutralize (Gradation) with a 'factive' view of infallibility, according to which the infallible justification that one feels rests on the presence of the pain itself, however marginal (Fumerton 1995: 75–76, 2010: 38, Huemer 2006: 152, 2007: 44, Hasan 2013: 122, Steup 2016: 73, Tucker 2016: 43). As Fumerton (2010: 381–82) complains, this 'metaphysical' construal is epistemically irrelevant; it separates infallibility from its epistemic root.

For the presentation of the puzzle, we need not endorse a particular measure of 'possibilities for error'. The basic point is simply that infallible justifications are both absolute and gradable in the same epistemic sense. To be sure, the gradation puzzle differs from the usual problem of vagueness. It is not equivalent, for example, to the task of locating the evasive threshold of 'tall'. After all, tallness is not absolute as a vague property; it smoothly comes in degrees. In contrast, infallibility tolerates absolutely no possible error, so it resists gradation: either a justification absolutely removes every possibility for error, or it does not. The puzzle also does not presuppose access internalism. While acknowledging less-than-perfect assurance may deprive Cartesians of full intellectual assurance (see Ballantyne 2012), the gradation puzzle does not assume that the agent recognizes the purported epistemic gradation.

Now, one might object that the conflict is illusory. The epistemic absoluteness of infallibility will indeed be safe if the gradation is not epistemic. For example, Fumerton's recent approach interprets the gradation as semantic:

> there are at least subtle differences between your thought of red and my thought of red, and further that my thought of red is almost certainly subtly unstable through time ... And at a time, as the correspondence between a thought and a fact is very weak, it does seem to me that one acquainted with the correspondence is often making more of a *decision* than a discovery with respect to whether or not the relevant thought is true. (Fumerton 2016: 243, 2018: 4680)

This proposal is designed to improve Fumerton's acquaintance theory, but we need not delve into details to appreciate its idea. At first glance, a person's justification that he feels pain weakens when his painful sensation subsides towards the borderline of 'pain'. According to Fumerton, this indeterminacy is essentially semantic. In near-borderline cases, what the person does is decide on the range of 'pain', rather than discovering any truth about pains. In referring to his sensation as 'painful', he acquires infallible justification that he feels pain. Alternatively, in deciding that his sensation is 'not painful', the person is infallibly justified that he does not feel pain. Linguistic behaviour has multiple norms, of course, and we cannot modify the range of a word at will. Nevertheless, when our thought about a term is unstable, determining its range is presumably within our power. As a result, what appeared to be epistemic uncertainty turns out to be a semantic issue. The gradation in justificatory force concerns the indeterminacy of how to use a term, not how well one perceives an object. Provided that we follow the custom of distinguishing semantic indeterminacy from epistemic uncertainty (BonJour 1997, Alston 1983), (Gradation) can be rejected.

Fumerton's view is prima facie plausible. Infallible justification would be perfectly absolute *if* properties such as pain had clear-cut boundaries. The best explanation is perhaps that the source of the gradation is semantic. To further support the semantic approach, consider slight variations in paradigmatic instances of a vague property. When motivating the puzzle, we assumed that two people can both be infallibly justified regarding their pains, while the one with stronger pain is nonetheless more justified. Fumerton's proposal, however, reminds us that severing the pain beyond a certain level may not enhance its likelihood of being 'pain'. Indeed, once a person is already infallibly justified due to sharp pain, intensifying the pain to higher degrees is unlikely to make additional epistemic contributions. Thus, in addition to working on cognitions of near-borderline cases, the semantic view also applies to the variations from paradigmatic instances of a property.

Despite its plausibility, the semantic approach cannot succeed. To see why, consider the following images:

Image A_3

Image A_{12}

Most people can infallibly tell that A_3 has three speckles. While the image may lack physical reality due to the possibility of illusion, our numerical justification for the *appearance* of A_3 cannot be mistaken. In contrast, we usually lack an infallible justification for A_{12}. For most people, counting is indispensable to determine that A_{12} has twelve speckles. Since counting involves memory, which can be misleading, the numerical truth of A_{12} is not transparent. As the well-known 'problem of the speckled hen' shows, we typically lose numerical certainty when looking at images with large numbers of speckles (see e.g. Chisholm 1942).

Nevertheless, we can learn to subitize A_{12} without intermediate steps. Suppose that an agent, Nicole, begins by identifying two groups of six speckles – for example, one group on the left and the other on the right – and eventually comprehends that A_{12} has twelve speckles. Nicole's resultant perception no longer rests on memory. She becomes capable of directly grasping the numerical truth of A_{12}. This assumption is not unrealistic. After all, A_{12} is not extremely complex; grasping its numerical truth should be possible when one is sufficiently familiar with this image. Thus, as a result of her exercise, Nicole's justification for A_{12} is infallible. She cannot be wrong about its number of speckles given her clear perception of this truth. Meanwhile, Nicole trivially retains the ability to subitize A_3. Therefore, she can successively acquire infallible justifications about the number of speckles in A_3 and A_{12}. When this happens, the former justification can still be stronger than the latter. A_3 is significantly simpler than A_{12}, so its numerical truth can be clearer than that of A_{12}. From Nicole's subjective viewpoint, she is more likely to be mistaken about A_{12} than she is about A_3.

The gradation issue resurfaces. How can numerical justification be infallible if it varies in strength? Nicole's case does not fit the semantic decision approach. Unlike colour or pain, the numerical truth of an image is not vague. Either A_{12} has twelve speckles or it does not. There are imprecise truths about A_3 and A_{12}, of course – for example, A_3 has 'more or less three speckles', and A_{12} has 'about ten speckles'. But this observation makes little room for the semantic view. In thinking of 'three' and 'twelve', Nicole's justifications about A_3 and A_{12} are extremely precise. The superior force of her justification about A_3 does not pertain to semantic determinacy. (Absolute) remains troubled by (Gradation). Infallibility is gradable in an irreducibly epistemic sense.

To defend our point, notice first that Nicole's infallible justification for A_{12} has tenable metaphysical grounds. Objectors may take Nicole as acquainted with only the determinable of 'having many speckles'. Proponents of the adverbial theory can similarly insist that Nicole is appeared to 'many-speckledly' but not 'twelve-speckledly'. In either case, Nicole's conscious states would not involve the truth-maker of the proposition that A_{12} has twelve speckles. These attacks, however, misrepresent our scenario. Given sufficient practice, Nicole can perceive the determinate property of having twelve speckles. She can also be appeared to twelve-speckledly on the adverbial theoretical account. Thus, our conclusion holds regardless of how we explain Nicole's sensory states.

The infallibility of Nicole's justification for A_{12} is also epistemically defensible. Objectors might regard Nicole's cognitive process as too complex to be individuated as a *single* act, but this issue poses little threat. The underlying psychology per se makes no difference because infallibility is grounded in phenomenal characters. What counts is the presentation of the numerical truth in Nicole's conscious mind, not the simplicity of her underlying cognitive process. It is noteworthy that, in making this point, we need not dismiss the externalist constraints of justification. To be sure, Nicole cannot reliably 'subitize twelve-speckled images' – she might be at a loss if the speckles are arranged differently than in A_{12} (see Zhang 2016). Nicole may even have to repeat her practice for each token of A_{12}. However, every time she succeeds in subitizing A_{12}, her local cognitive process is ultra-reliable. Externalists are thereby in no position to suspect the infallibility of Nicole's justification.

Another objection alleges that, once Nicole completes the practice, her justifications about the two images are equally clear. Behind this objection is the factive view that a justification is absolute whenever it metaphysically entails the relevant truth-maker. For our case, however, this proposal ignores a phenomenal difference between Nicole's perceptions of A_3 and A_{12}. When we fix Nicole's attention, abilities, etc., the numerical truth of A_3 can still be 'clearer' than that of A_{12}. Since infallibility is essentially phenomenal, this superior clarity immediately induces a stronger justificatory force. Insisting otherwise will again cut infallibility from its epistemic root.

What we have is not an isolated case. The same conclusion can be reached from any scenario in which infallible justifications have precise contents but nonetheless vary in strength. To borrow BonJour's (1997: 119) example, one can be infallibly justified with respect to '$2 + 2 = 4$' and '$2^5 - 5 = 3^3$'. Through rational intuitions of abstract truths, such justifications can be as infallible as sensory introspections in the Cartesian framework. The proposition '$2^5 - 5 = 3^3$' is admittedly complex, but one might directly grasp its truth after sufficient exercise. Meanwhile, '$2 + 2 = 4$' can remain clearer than '$2^5 - 5 = 3^3$', despite one's intuitions of both mathematical truths. If obliged to reject either equation, one should preserve '$2 + 2 = 4$' instead of '$2^5 - 5 = 3^3$'. This gradation in evidential force does not concern semantic vagueness. Mathematical equations are perfectly precise.

Notably, our cases about numerical and mathematical truths are not epistemically irrelevant. Infallibility serves to fulfil the Cartesian project of finding a secure epistemic basis. For such a basis to sustain a wide range of knowledge, it must include numerical justification and rational intuition. Sensations cannot be the only object of intellectual assurance (see Sosa 2009: 27–29). Admittedly, advocates of the semantic decision view may opt for a disjunctive approach, insisting that the gradation of infallible justification for *sensations* is nonetheless semantically reducible. This rejoinder is perhaps consistent, but it lacks sufficient explanatory power. Both the infallible justifications about sensations and mathematics appear to present the truths to our minds. Both appear to be gradable in strength. Therefore, the disjunctive strategy will not only divide the nature of infallibility but also disassociate it from its phenomenal ground. To compensate for this loss, advocates of this approach must offer a principled account of why some infallible justifications are epistemically gradable, while others are not. Without such an account, the disjunctive reply is ad hoc against our attack.

We have formulated the gradation puzzle against infallibility, but what is the puzzle's target range? Cartesians may complain, for example, that infallibility is an overly strong characterization of the desired epistemic state. Fumerton often construes his theory as targeting 'non-inferential justification', which need not be infallible. A person can plausibly have a fallible non-inferential justification that he feels pain even when his pain is mild like an itch. This justification is fallible because it can mislead, but it is not inferred from any other belief. Because 'non-inferential' concerns only the structure of one's reason and not its strength, it is compatible with gradations in evidential force.

The mere structure of being 'non-inferential', however, cannot satisfy Cartesian expectations. In focusing on non-inferential justifications, what Fumerton more precisely seeks is 'intellectual assurance'. Intellectual assurance is often taken as infallible, but it also refers to a justification that is '*as good as it gets*' (Fumerton 2016: 248, 2018: 4680). Thus construed, intellectual assurance requires justificatory force but need not be infallible. For example, when presented with a slightly curved line, a person's justification for describing it as straight is fallible, but it is perhaps ‘as good as it gets’ (Tucker 2016: 44). Unfortunately, such fallibility does not exempt intellectual assurance from the gradation problem. On a plausible reading, if a state is as good as it gets, then it cannot be improved. Nicole’s numerical justification for A_{12}, however, is improvable even when she subitizes the image. Provided that her numerical justification for A_{12} is weaker than that for A_3, there is room for her to strengthen the former. Consequently, Nicole’s numerical justification for A_{12} both is and is not ‘as good as it gets’. Again, this conflict is not semantically resolvable. Throughout the improvement process, Nicole’s relevant justifications invariably support the proposition that A_{12} has twelve speckles. Thus, we can reformulate the gradation puzzle for intellectual assurance as follows:

(Absolute$_{\text{Best}}$) The intellectual assurance for *P* is the best justification that a person can possibly have to support *P*.

(Gradation$_{\text{Best}}$) A person’s intellectual assurance for *P* can be better than another of his intellectual assurances for *P*.

Unlike (Gradation), (Gradation$_{\text{Best}}$) does not compare intellectual assurances for different propositions. Without further clarification, the question of whether a justification is ‘better than’ another arguably makes sense only when both target the same proposition. Since Nicole’s best numerical justification for the very same image A_{12} is improvable, (Gradation$_{\text{Best}}$) is true, and it counters (Absolute$_{\text{Best}}$).

In addition to ‘as good as it gets’, one can adopt an ‘appearance’ account of intellectual assurance. An intellectual assurance would accordingly be what *appears* to tolerate no possibility of error. Thus understood, intellectual assurance remains fallible: what appears to eliminate all possible errors need not truly do so. Unsurprisingly, the conflict between absoluteness and gradation can enter the realm of appearance, as the following pair of statements illustrates:

(Absolute$_{\text{Appearance}}$) The intellectual assurance for *P* appears to eliminate every possibility that *P* is false.

(Gradation$_{\text{Appearance}}$) An intellectual assurance for *P* can appear to leave more possibilities that *P* is false than another intellectual assurance for *P′*.

(Absolute$_{\text{Appearance}}$) and (Gradation$_{\text{Appearance}}$) are incompatible. A state cannot both appear to eliminate all possibilities of error and appear not to do so. This conflict is again irreducible to semantic indeterminacy. For Nicole, the numerical justifications about A_3 and A_{12} both appear to eliminate all of their possible errors. Nevertheless, her numerical justification for A_3 appears to eliminate more.

In conclusion, we have presented the gradation puzzle of intellectual assurance and explained why semantic vagueness cannot resolve it. Intellectual assurance is supposed to be absolute, but it is also gradable in an irreducibly epistemic sense. While we initially formulated the puzzle against infallibility, it indifferently threatens fallibilist construals of intellectual assurance.[1]

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Corrigendum to: Why punitive intent matters

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In the originally published version of this article, several errors were introduced but have now been amended. Footnote 2 should have been deleted entirely, but only the footnote indicator in the text was removed. This text has now been deleted so the following footnotes have been amended. The footnote text for what is now footnote 7 was omitted and this has now been added. OUP apologises for these errors.

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